Exercise 11

Use the definition of continuity and the properties of limits to show that the function is continuous at the given number a.

$$f(x) = (x + 2x^3)^4, \quad a = -1$$

Solution

By definition, a function is continuous at a number a if

$$\lim_{x \to a} f(x) = f(a).$$

Evaluate the function at $x = -1$.

$$f(-1) = [(-1) + 2(-1)^3]^4 = (-1 - 2)^4 = 81$$

Calculate the limit as x approaches -1 using the limit laws.

$$\begin{aligned}
\lim_{x \to -1} f(x) &= \lim_{x \to -1} (x + 2x^3)^4 \\
&= \lim_{x \to -1} (x + 2x^3)(x + 2x^3)(x + 2x^3)(x + 2x^3) \\
&= \left[\lim_{x \to -1} (x + 2x^3)\right]\left[\lim_{x \to -1} (x + 2x^3)\right]\left[\lim_{x \to -1} (x + 2x^3)\right]\left[\lim_{x \to -1} (x + 2x^3)\right] \\
&= \left[\lim_{x \to -1} (x + 2x^3)\right]^4 \\
&= \left[\lim_{x \to -1} (x) + \lim_{x \to -1} (2x^3)\right]^4 \\
&= \left[\lim_{x \to -1} (x) + 2 \lim_{x \to -1} (x^3)\right]^4 \\
&= \left[\lim_{x \to -1} (x) + 2\left(\lim_{x \to -1} x\right)\left(\lim_{x \to -1} x\right)\left(\lim_{x \to -1} x\right)\right]^4 \\
&= \left[\lim_{x \to -1} (x) + 2\left(\lim_{x \to -1} x\right)^3\right]^4 \\
&= [(-1) + 2(-1)^3]^4 \\
&= (-1 - 2)^4 \\
&= 81
\end{aligned}$$

The condition in the definition is satisfied, so $f(x) = (x + 2x^3)^4$ is a continuous function at $a = -1$.